Permeability of periodic porous media

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The permeability of the two-dimensional periodic arrays of cylinders is obtained numerically as a function of the dimensionless wave number kD, where k is the wave number based on the distance between particles in the streamwise direction and D is the diameter. To isolate the kD dependence, D and the porosity are held fixed. The latter is achieved by making the product of distance between particles in the cross-stream and streamwise directions constant. The numerical results show that the permeability increases with kD, but the increase is not monotonic. In particular, the permeability decreases for $\sim 5 < kD < \sim 7.7$, and becomes locally minimum at $kD \approx 7.7$. This value of kD is significant because it is the smallest wave number for which the streamwise area-fraction spectrum is zero. For $kD < 5$ and $7.7 < kD < 11$, the permeability increases with kD. Our numerical simulations also show that for $kD \approx 7.7$ the pressure distribution in the cross-stream direction is relatively flat which again is a consequence of the fact that the area-fraction distribution in the flow direction is approximately constant. [S1063-651X(99)04801-1]

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I. INTRODUCTION

The direct numerical simulation capability for investigating the porous media flows is important in a number of applications. The objectives in these applications may range from determining the flow field for the given velocity and pressure boundary conditions to designing a porous medium with the desired flow properties. An example of the former is the direct numerical simulation of underground flows in the oil recovery applications. Examples of the latter include many applications where the objective is to design the optimal porous medium geometry for trapping or absorbing fluids. It is noteworthy that even though the flow field in a porous medium varies at length scales comparable to the size of particles, in many applications a detailed knowledge of these particle scale variations is not required. Instead, an averaged macroscopic response is sufficient. A detailed knowledge of the particle scale variations, however, can be helpful in designing the optimal porous medium for specialized applications.

A macroscale model such as Darcy's law is extremely useful because it allows us to predict the macroscopic flow field without solving the detailed fluid flow problem. Darcy's law states that the volume averaged superficial fluid velocity U is proportional to the pressure gradient:

$$U = \kappa \frac{\nabla p}{\mu}, \tag{1.1}$$

where μ is the viscosity, p is the pressure, and κ is the effective permeability tensor for the medium. The permeability tensor κ for a medium depends on several factors, including the porosity and the particle size distribution. Several correlations have been developed to relate the permeability of a porous medium to these distributions (see [1–4]), and the references listed in these papers).

The permeability of a porous medium also depends on the microstructure or the relative arrangement of particles. In fact, analysis based on Darcy's law works well only when the assumptions used for driving it are valid; i.e., the medium is random, completely wet (the surface tension effects are not present), and the fluid is Newtonian [1,5–7]. A porous medium, in general, may not be random, but have a distinct microstructure, i.e., the particle arrangement contains a preferred direction for the flowing fluid. This may be the case when the particles are arranged in a systematic pattern, as is the case for many man-made porous materials. To apply Darcy's law in such cases, the macroscopic permeability should be determined by accounting for the microstructure [8,10].

It is possible to numerically estimate the macroscopic permeability of a microstructured porous medium by including its key microstructural features in the computational domain. There are several direct numerical studies where the permeability of a collection of periodically arranged particles is determined numerically (see [3,11,12], and the references listed in these papers). For example, it was shown in [11,12] that the permeability of the face-centered-cubic (fcc), body-centered-cubic (bcc), and simple-cubic (sc) lattices of spheres—with the same porosity and D—are different. In other words, the arrangement of particles relative to the flow direction is important in determining the permeability.

In this paper we study this dependence of permeability on the microstructure by continuously varying kD (or the area-fraction distribution) along the flow direction for a two-dimensional porous medium. Our simulations show that for a given porosity and particle size, the permeability depends on the wavelength λ or the distance between the particles in the flow direction [see Fig. 1(a)]. The diameter D and λ can be used to define a dimensionless wave number kD for the porous medium, where $k = 2\pi/\lambda$. For small kD, the permeability increases with increasing kD, but this increase is not monotonic. The permeability starts to decrease at $kD \approx 5$ and reaches a local minimum at $kD \approx 7.7$. As we will discuss in the next section, the kD value at this local minimum of permeability is significant because it is the smallest dimensionless wave number for which the area-fraction spectrum is zero. Thus the changes in permeability and area-fraction distribution with kD are related.

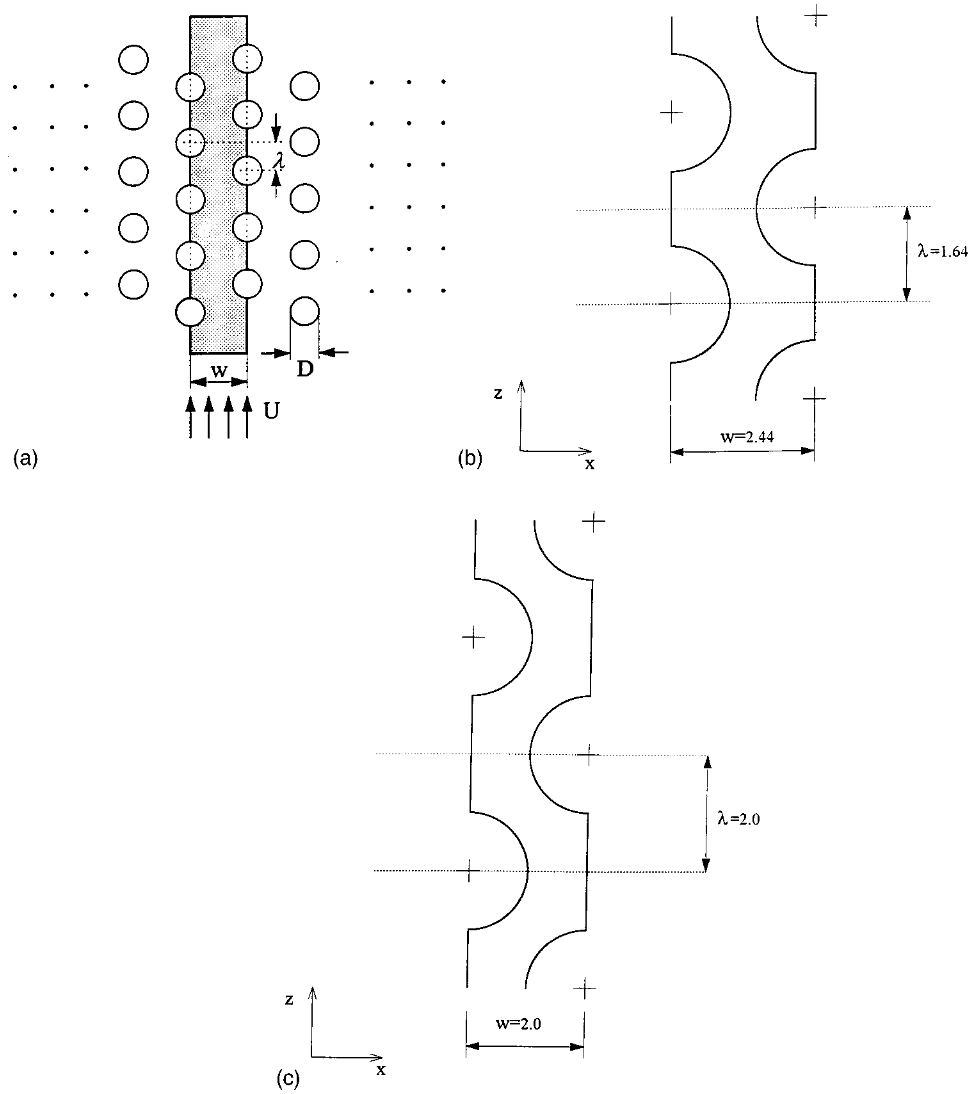

FIG. 1. (a) A schematic of a periodic array of cylinders is shown. The interparticle distance in the z direction is λ and in the cross-stream direction is w. The shaded area is a typical computational domain for our simulations. (b) The porous medium for $\lambda = 1.64$, (c) the porous medium for $\lambda = 2.0$.

II. PERMEABILITY AND AREA-FRACTION DISTRIBUTION OF A PERIODIC POROUS MEDIUM

In this study we will assume that the particles (or cylinders) are arranged periodically in the space as shown in Fig. 1(a). Therefore away from the entrance and the exit the velocity field is periodic, and the pressure drop for each period cell is constant. This constant pressure drop for a period cell determines the permeability of the porous medium.

A typical computational domain used in our simulations is shown in Fig. 1(b). The domain is specified in terms of two parameters: the distance λ used for arranging particles in the flow direction and the width w. To ensure that the solids fraction $0.5\pi R^2/\lambda w$ is constant, the product λw was held fixed at 4 and the particle radius $R=1$. Thus the solid fraction for our simulations is 0.393.

To define the area fraction on the planes perpendicular to the flow direction—assumed here to be the z axis of the coordinate system—we will assume that the particles are cylinders of radius R. The number density distribution is denoted by $N(z)$. Let the area fraction $\phi_a(z)$ be the fraction of the plane z covered by the particles. The following expression for ϕ_a can be obtained by adding the areas of intersections of $N(z+x)$ particles that have centers at a distance x from the plane z:

$$\phi_a(z) = \int_{-D/2}^{D/2} N(x+z) 2\sqrt{D^2/4 - x^2}\, dx. \qquad (2.1)$$

Note that the integration limits are from $-D/2$ to $D/2$, as only the particles that are at a distance less than R from the plane z have a nonzero area of intersection (see [13] where the area fraction was obtained for the spherical particles).

Assuming that $N(z)$ is in the Fourier transform class, the Fourier transform of the above expression can be obtained using the convolution-multiplication theorem

$$\phi_a(k) = D^2 \Theta(kD) N(k), \qquad (2.2)$$

where k is the wave number, $\Theta(kD) = \int_{-1/2}^{1/2} 2\sqrt{\frac{1}{4} - x^2}\, e^{ikDx} dx$ is the blockage function, $\phi_a(k)$ is the Fourier transform of $\phi_a(z)$, and $N(k)$is the Fourier transform of $N(z)$. The graph of $\Theta(kD)$ is shown in Fig. 2. From

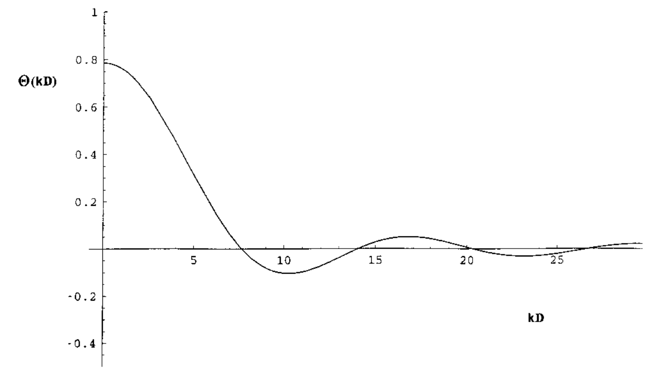

FIG. 2. $\Theta(kD)$ is plotted as a function of kD.

this figure we note that the set of zeros of $\Theta(kD)$ is $kD=7.663,14.031,20.347,\ldots$. Expression (2.2), therefore, implies that the dimensionless wave numbers kD, for which $\Theta(kD)$ is zero, are *blocked*, i.e., are missing in the spectrum of the area fraction.

To understand the physical significance of $\Theta(kD)$ we note that if $N(k)$is nonzero only for the wave numbers for which $\Theta(kD)$ is zero, the area available to the flowing fluid in the z direction will be constant, as in this case $\phi_a(k)=0$ for all $k>0$. On the other hand, if $\Theta(kD)$ is nonzero for $N(k)\neq 0$, the area fraction will vary with z. For example, for the periodic porous medium shown in Fig. 1(b), $N(k)=\Sigma_n\delta(k-2\pi n/\lambda)$, where n is an integer and $\lambda/R=2\pi/kR=1.64$ is the distance between particles in the flow direction. In this case, for the first nonzero mode of $N(k)$, i.e., $kD=7.66$, $\Theta(kD)$ is zero, and thus this mode does not cause any variation in the area available to the fluid. The area fraction—i.e., the convolution of Eq. (2.1)—is, however, not exactly constant because only the first zero of $\Theta(kD)$ coincides with the first nonzero term of $N(k)$. For the higher order modes of $N(k)$, $\Theta(kD)$ is nonzero. But, since $\Theta(kD)$ is small for the higher order modes of $N(k)$, the area fraction varies—but only slightly—with z. For $\lambda=2$ ($kD=6.28$), on the other hand, since $\Theta(6.28)\neq 0$, the area fraction varies substantially with z [see Fig. 1(c)].

It is noteworthy that for the sc, fcc, and bcc lattices

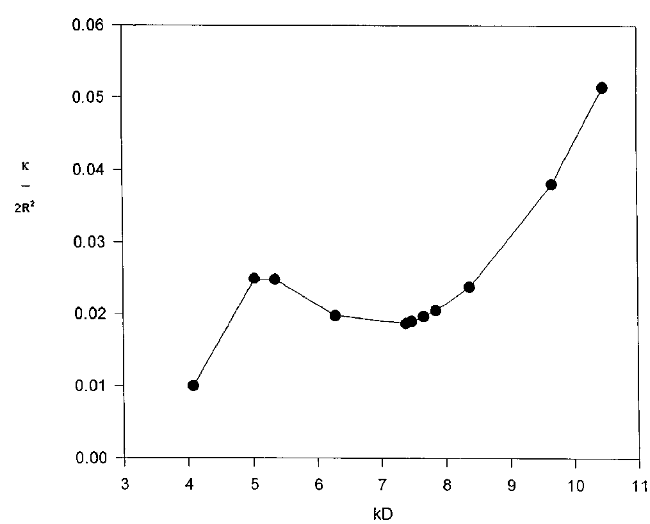

FIG. 3. The numerically computed permeability κ/R^2 is shown as a function of kD.

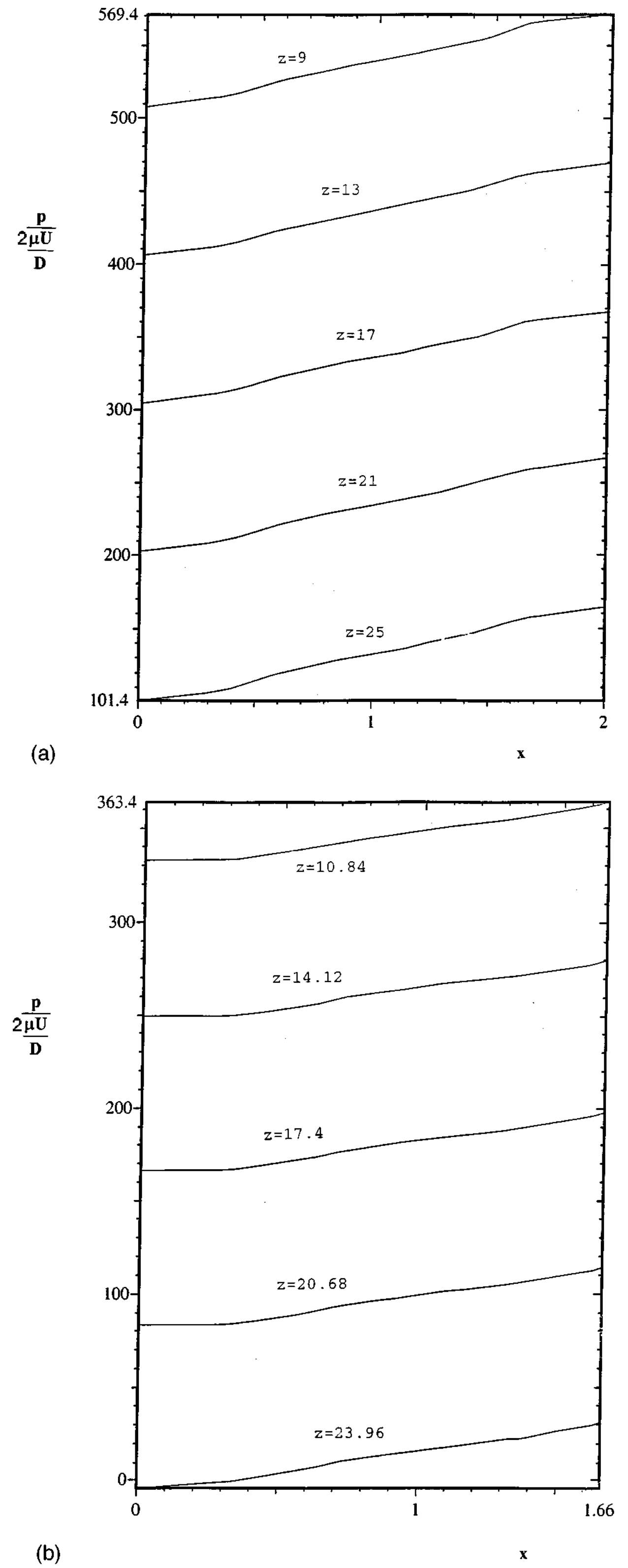

FIG. 4. The dimensionless pressure distributions in the cross-section direction are shown at five different z locations. (a) $\lambda=2.0$, (b) $\lambda=1.64$.

with the same porosity, the kD value along the flow direction is different. For example, when the solid fraction is 0.45 the kD values for the sc, fcc, and bcc lattices are 5.97, 6.98, and 9.48, respectively. As noted before, the past numerical studies show that the permeability of the sc lattice is smaller than that of the fcc lattice, but the permeability of the fcc lattice is larger than that of the bcc lattice [8,9]. Thus the permeability changes nonmonotonically with kD. The permeability for other values of kD is not known, as in these studies only the above three cubic lattices were studied. This nonmonotonic variation of permeability for spherical particles is interesting, and needs further investigation.

III. PROBLEM STATEMENT AND BOUNDARY CONDITIONS

Our objective is to numerically simulate the periodic porous media flows, and use the simulation results both for estimating the effective permeability and for understanding the microscale flow features. The Navier-Stokes and continuity equations are solved using the finite-element method in the periodic computational domains using the boundary conditions described below. The Reynolds number is assumed to be zero. The details of the numerical method are given in Ref. [14].

The following boundary conditions are applied at the boundaries of computational domain. At the inlet of the computational domain the incoming superficial velocity U is specified. Along the sides of the computational domain periodic boundary conditions, i.e., $u=0$, $\partial v/\partial x=0$, are applied, where u is the x component of velocity and v is the y component of velocity. At the exit of the computational domain the no-traction boundary condition is applied, and the no-slip boundary condition is imposed at the surface of particles.

IV. RESULTS AND CONCLUSIONS

For our simulations the permeability κ along the z direction is calculated by using the definition of Darcy's law:

$$\kappa=\frac{\mu U}{\Delta p/\lambda}, \tag{4.1}$$

where Δp is the pressure drop over the distance λ, μ is the viscosity, and U is the superficial velocity. As noted before, for our simulations the porosity is 0.607, and the viscosity and the superficial velocity are held fixed.

We first note that the permeability of a channel of width w varies as w^2. Since for our simulations $\lambda w=4$ and the wave number k is inversely related to λ, the width of an equivalent flat channel increases with kD. The computed values of the permeabilities are shown in Fig. 3 as a function of the dimensionless wave number kD. From this figure we note that for $kD<5$, as expected, the permeability increases with increasing kD. But, for $5<k<7.7$ the permeability decreases with kD. The permeability reaches a local minimum at $kD \approx 7.7$. After reaching this local minimum, the permeability increases with increasing kD. The calculations were carried out for kD up to 11. Note that for $kD\approx 7.7$, $\Theta(7.7)\approx 0$, i.e., this value of kD is missing in the area-fraction spectrum (see Sec. II). This shows that the area-fraction distribution along the flow direction plays a role in determining the permeability.

Another interesting feature for $kD=7.7$ is that the pressure distribution in the cross-stream direction is relatively flat [see Fig. 4(a)]. For $kD=6.28$, on the other hand, the pressure varies substantially in the cross-stream direction [see Fig. 4(b)]. This again is related to the area-fraction variation in the flow direction. In particular, when the area available to the flow is constant or varies relatively slowly in the streamwise direction the pressure distribution in the cross-stream direction is relatively flat.

Finally, we note that for our periodic porous medium $N(k)$ contains higher order modes—i.e., in addition to the primary mode at $k=2\pi/\lambda$—for which $\Theta(kD)$may not be zero. For example, even for $\lambda/R=1.64$ the area fraction is not exactly constant in space. But, since $\Theta(kD)$ is small for the higher order modes, they lead to only small variations in the area fraction along the streamwise direction. Furthermore, as $\Theta(kD)$ decreases with increasing kD, the dependence of permeability on the area fraction decreases with kD, and the permeability approaches the value for an equivalent flat channel.

ACKNOWLEDGMENT

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[1] R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena* (Wiley, New York, 1960).
[2] J. J. L. Higdon and G. D. Ford, J. Fluid Mech. **308**, 341 (1996).
[3] G. W. Jackson and D. F. James, Can. J. Chem. Eng. **64**, 364 (1986).
[4] J. E. Drummond, and M. I. Tahir, Int. J. Multiphase Flow **10**, 515 (1984); **10**, 540 (1984).
[5] D. F. James and D. R. Mclaren, J. Fluid Mech. **70**, 733 (1975).
[6] A. B. Metzner and R. J. Marshall, Industrial Eng. Chem. Fundamentals **6**, 393 (1967).
[7] S. Rodriguez, C. Romero, M. L. Sargenti, A. J. Muller, A. E. Saez, and J. A. Odell, J. Non-Newtonian Fluid Mech. **49**, 63 (1993).
[8] R. E. Larson and J. J. L. Higdon, Phys. Fluids **1**, 38 (1986).
[9] W. L. Olbricht, Annu. Rev. Fluid Mech. **28**, 187 (1996).
[10] T. D. Papathanasiou, Compos. Sci. Technol. **56**, 1055 (1996).
[11] H. Hasimoto, J. Fluid Mech. **5**, 317 (1959).
[12] A. A. Zick and G. M. Homsy, J. Fluid Mech. **115**, 13 (1982).
[13] P. Singh and D. D. Joseph, Int. J. Multiphase Flow **21**, 1 (1995).
[14] P. Singh, Ph. Caussignac, A. Fortes, D. D. Joseph, and T. Lundgren, J. Fluid Mech. **205**, 553 (1989).